RESEARCH STATEMENT

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One of the crowning achievements of the Grothendieck school of algebraic geometry is the following consequence of the Grothendieck–Riemann–Roch theorem [BS58], [BGI71]: if X is a smooth projective variety over a field k, then there is a natural isomorphism of $\mathbf{Q}$-algebras.

$$\mathrm{K}_0(\mathrm{X})_{\mathbf{Q}} \xrightarrow{\simeq} \mathrm{CH}^*(\mathrm{X})_{\mathbf{Q}}.$$

Here, $\mathrm{K}_0(\mathrm{X})$ is the (rationalization of) the Grothendieck ring of algebraic vector bundles on X, while $\mathrm{CH}^*(\mathrm{X})$ is the Chow ring of algebraic cycles on X up to rational equivalence. Philosophically, this result is striking for at least a couple of reasons:

(1) the left hand side is defined for a wider class of objects than just varieties; the K_0 group only depends on the the category of vector bundles on X and is defined more generally for categories with "exact sequences".
(2) The right hand side has a *grading* which corresponds to the codimension of the algebraic cycles on X which generate the individual Chow group $\mathrm{CH}^i(\mathrm{X})$.

The isomorphism above unites two perspectives on the *motivic* aspect of algebraic varieties: the commutative (right hand side) and the noncommutative (left hand side). The noncommutative perspective studies a variety by means of "linear algebraic" objects that live on the variety like the category of vector bundles on these varieties. These linear algebraic objects assemble into a category where one can do "linear algebra" and axiomatized under its modern incarnation, a "stable ∞-category." The commutative perspective studies a variety by means of "geometric" objects that live on a variety, such as algebraic cycles. These objects often equip an extra grading on the invariants — in the above example this is the grading by codimension present on Chow rings.

Much of my research is inspired by the interaction between commutative and noncommutative motives as exemplified by the Grothendieck-Riemann-Roch theorems. Some projects seek to import theorems or inspiration from one world to another, others seek to use one world to say something about the other. All of them use ideas and techniques from homotopy theory.

1. Research Summary

1.1. **Algebraic cobordism, Hilbert schemes and derived algebraic geometry.** The geometric origin of modern stable homotopy theory lies in the work of Thom and Pontryagin who established deep connections between the stable homotopy groups of spheres and framed manifolds [Tho54], bringing to bear the computational machinery of stable homotopy theory to questions about the geometry of manifolds. More recently Madsen and Weiss proved the celebrated Mumford conjecture on the moduli space of curves [MW07] by identifying the stabilized moduli space of curves with the Thom spectra of certain negative bundles; this result has since been vastly generalized and expanded by Galatius, Madsen, Tillman and Weiss [GMTW09]. In all these cases, one describes certain moduli spaces of geometric objects *up to cobordism* as the spaces appearing in a certain spectrum, at least after performing group completion.

A direct algebro-geometric analog of a cobordism is an $\mathbf{A}^1$-path in the moduli space of smooth, proper schemes, i.e., a smooth, proper morphism $f : \mathrm{W} \to \mathbf{A}^1$ defining a cobordism between the generic and the special fibers. This analogy, however, fails on the nose because of *transversality issues.* Nonetheless, in the world of stable motivic homotopy theory Voevodsky constructed **algebraic cobordism** MGL as a motivic spectrum by mimicking the formula in topology using universal vector bundles over Grassmanian schemes. Various workers have established the homotopy theoretic properties of this spectrum such as its universality [PPR08] and, after the characteristic of the ground field is inverted, its ring of coefficients [Hoy15]. Despite many attempts [LM07], [LP09], [LS16], a complete geometric description of MGL, however is still wanting.

In [EHK$^+$20b], joint with Hoyois, Khan, Sosnilo and Yakerson, I gave a new geometric interpretation of algebraic cobordism overcoming the attendant transversality issues. Let $\mathcal{FQS}\mathrm{m}^0$ denote the moduli stack of finite, quasismooth derived schemes of virtual dimension zero.

Theorem 1.1.1. [EHK$^+$20b] *Let k be a field, then $\Omega^{\infty}_{\mathbf{P}^1}\mathrm{MGL}$ is the motivic group completion of the stack of $\mathcal{FQS}\mathrm{m}^0$.*

In the same paper, we were also able to describe the spaces $\Omega^{\infty}_{\mathbf{P}^1}\mathrm{MGL}_{2n,n}$, representing degree n-MGL cohomology for $n > 0$ as the moduli of negative dimensional (precisely, dimension $-n$) quasi-smooth derived schemes. In another direction, we made Theorem 1.1.1 more concrete using Hilbert schemes. To describe this model, we used a locus inside the Hilbert scheme of points in $\mathbf{A}^{\infty}$, consisting of those points which are local complete intersections. This turns out to be a smooth (ind-)scheme which we denote by $\mathrm{Hilb}^{\mathrm{lci}}(\mathbf{A}^{\infty})$. A moving lemma, established additionally with Bachmann, gives us:

Theorem 1.1.2. [BEH$^+$19] *Over a field, we have a motivic equivalence*

$$\Omega^{\infty}_{\mathbf{P}^1}\mathrm{MGL} \simeq \mathrm{Hilb}^{\mathrm{lci}}(\mathbf{A}^{\infty})^+ \times \mathbf{Z}.$$

As an application of these results, in joint work with Bachmann [BE19], I gave short and conceptual proofs of theorems of Levine's [Lev08] which were conjectures of Voevodsky's [Voe02, Conjectures 2,3]. These (now-proved) conjectures result in the identification of the E_2-page of the motivic spectral sequence as Bloch's higher Chow groups. While Levine's proof used difficult moving lemma, we reduced the conjectures to a study of the birational geometry of Hilbert schemes.

1.2. **Transfers and norms in motivic homotopy theory and other contexts.** Theorem 1.1.1 relies on our previous work on the **motivic recognition principle** [EHK$^+$19] and [EHK$^+$20a] which solved the long-standing problem posed by Voevodsky on recognizing infinite loop spaces in motivic homotopy theory. A culmination of both papers state state that over a perfect field, the data of an effective (the motivic analog of connective) motivic spectrum over a perfect field is the same thing as (1) an $\mathbf{A}^1$-invariant Nisnevich sheaf of spectra with (2) coherent transfer maps along finite syntomic maps equipped with a trivialization of the cotangent complex.

One application of this, joint with with Khan [EK18], is a motivic analog of Grothendieck's "équivalence remarquable de catégories" [Gro67, Théorème 18.1.2]. We proved:

Theorem 1.2.1. [EK18] *If $f : \mathrm{X} \to \mathrm{Y}$ is a universal homeomorphism of schemes, then the functor $f^* : \mathbf{SH}(\mathrm{Y}) \to \mathbf{SH}(\mathrm{X})$ is an equivalence after inverting residual characteristics.*

Another direction that that I have recently pursued is of a more foundational nature. Various category-valued functors in homotopy theory can be equipped with **multiplicative pushforwards**. For example, Bachmann and Hoyois have constructed in [BH18] multiplicative pushforwards along étale morphisms for the functor **SH** which categorifies various norm constructions in K-theory and Chow groups, while Hill, Hopkins and Ravenel [HHR16] constructed norm maps along maps of finite groups for equivariant spectra. In a project with Haugseng, we associated to an $\infty-$category with two suitable classes of morphisms — one class for additive pushforward and another for multiplicative pushforward along the other, its $(\infty, 2)$-category of **bispans**. We then proved a universal property for this category [EH20], which lets us solve the coherence problem posed by these multiplicative norms constructions and promote the above functors to ones out of bispans; this expands the universal property for categories of spans studied by Gaitsgory and Rozenblyum [GR17] and elaborated by Macpherson [Mac20]. As a result, we promote the formation of motivic and equivariant spectra, as well as perfect complexes, to functors out of categories of bispans.

1.3. **Étale motivic homotopy theory.** Starting with Thomason in [Tho85], it is known that inverting a certain "Bott element" in algebraic K-theory results in étale descent. This result was extended to algebraic cobordism in joint work with Levine, Spitzweck and Østvær [ELSØ17]:

Theorem 1.3.1. [ELSØ17] *Let X be a noetherian scheme of finite dimension with a uniform bound on cohomological dimension of its residue fields. Let $\mathrm{MGL}^{\text{ét}}$ be étale version of MGL. Then, after p-completion where p is invertible in X, there is a canonical isomorphism*

$$\widehat{\mathrm{MGL}}_p(\mathrm{X})[\tau^{-1}] \simeq \widehat{\mathrm{MGL}^{\text{ét}}}_p(\mathrm{X}),$$

where τ is a certain canonical defined element in the homotopy groups of $\widehat{\mathrm{MGL}}_p$.

More recently, with Østvær and Bachmann, I improved the result above by proving a similar result for the motivic sphere spectrum $\mathbf{1}$, and under less restrictive hypotheses.

Theorem 1.3.2. [BEØ19] *Let* X *be a noetherian finite dimensional base scheme and let* p *prime which is invertible in* X *such that the* virtual cohomological dimension *is uniformly bounded for all its residue fields Then, after* p*-completion and* ρ*-completion, where* p *is invertible in* X*, there is a canonical isomorphism*

$$\widehat{\mathbf{1}}_p(\mathrm{X})[\tau^{-1}] \simeq \widehat{\mathbf{1}^{\text{ét}}}_p(\mathrm{X}),$$

where τ is a certain canonical defined element in the homotopy groups of $\widehat{\mathbf{1}}_p$.

Crucial to proving this theorem, we proved a derived version of Voevodsky's conjecture [Voe02, Conjecture 13] concerning the convergence of the slice spectral sequence for fields of finite virtual cohomological dimension.

1.4. ***b*-motivic homotopy theory.** The b-topology, introduced by Scheiderer, is a Grothendieck topology relevant for schemes over ordered fields [Sch94]. Its category of sheaves amalgamate together the étale and real étale topology of said schemes, and enjoys excellent formal properties. In joint work with Shah, I addressed the problem of understanding motivic spectra in the b-topology by proving a rigidity theorem in the style of [Ayo14, CD16, Bac18].

Theorem 1.4.1. [ES19] *If k is real closed field, then there is an equivalence of ∞-categories:*

$$\mathbf{SH}_b(k)_{\mathrm{prof}} \simeq \mathrm{Spt}^{\mathrm{C}_2}_{\mathrm{prof}},$$

where 1) $\mathbf{SH}_b(k)$ *denote motivic spectra in the b-topology, 2)* $\mathrm{Spt}^{\mathrm{C}_2}$ *denote (genuine)* C_2*-equivariant spectra (modeled by Spectral Mackey functors as in* [Bar17]*) and 3)* prof *indicate the full subcategories of profinitely-completed objects.*

This equivalence persists for suitable schemes, in place of Spec k where the right-hand-side is replaced with a category "parametrized C_2-spectra" which we constructed by topos-theoretic methods. Two novel aspects of Theorem 1.4.1: 1) we identify the C_2-Tate construction in equivariant homotopy theory purely algebro-geometrically and 2) the right notion of rigidity for b-motivic spectra turns out to involve genuine C_2-equivariant homotopy theory as opposed to just spectra or complexes which made the problem more subtle.

1.5. **Excision in motivic homotopy theory.** A cartesian diagram of schemes

$$\begin{tikzcd} \mathrm{X}' \arrow[r] \arrow[d] & \mathrm{X} \arrow[d, "p"] \\ \mathrm{Y}' \arrow[r, "k"] & \mathrm{Y}. \end{tikzcd} \tag{1.5.1}$$

where p is an affine morphism and k is a closed immersion is called a **Milnor square**. If $\mathcal{F}$ is an an invariant of schemes, say valued in spectra or chain complexes, then we say that $\mathcal{F}$ has **Milnor-excision** if it takes the square above to a cartesian square. This property has proved useful in recent years. For example, a pro-version of Milnor excision was one crucial step in proving Weibel's conjecture on negative K-theory [KST17, LT19]. A mechanism to prove Milnor excision was given by Bhatt and Mathew using their **arc topology** [BM18].

In joint work with Hoyois, Iwasa and Kelly, we were able to formulate a different framework to address the problem of Milnor excision for motivic spectra. These *do not* necessarily satisfy arc descent. Instead, we defined a new Grothendieck topology called the **cdarc topology** for which we could prove descent for these cohomology theories and still prove Milnor excision and formal gluing.

Theorem 1.5.2. [EHIK20a, EHIK20b] *Any cohomology theory represented in the stable motivic category satisfies Milnor excision.*

Theorem 1.5.2 is, the first theorem of its kind for a general motivic spectrum which is not homotopy K-theory or étale cohomology. For example, one can plug in the motivic cohomology spectrum and get excision result for the Chow groups of algebraic cycles.

1.6. **Derived algebraic geometry methods in derived categories.** A key notion in understanding the geometry of schemes via a noncommutative viewpoint is the notion of semiorthogonal decompositions which breaks down the category of perfect complexes in terms of smaller constituent pieces and gluing data between them. The simplest example of this is the decomposition of $\mathbf{Perf}(\mathbf{P}^1)$ due to Beilinson in [Bei78] in terms of $\mathcal{O}$ and $\mathcal{O}(1)$. In joint work with with Antieau, I systematically studied the problem of descending semiorthogonal decompositions in arithematically interesting situations. Inspired by work of Bernardara on Severi-Brauer schemes, Bernardara-Auel [Ber09] on twisted forms of Del Pezzo surfaces [AB18] and Ballard-Duncan-McFaddin on arithmetic toric schemes [BDM17], we gave a uniform explanation of these examples by constructing a (derived) stack which parametrizes families of semiorthogonal decomposition [AE19]. We not only recovered and unified the descent results of these previous papers, but also set up a theory of “characteristic classes” which defines obstructions to descending semiorthogonal decompositions.

1.7. **Trace methods and topological cyclic homology (TC).** More recently, I have studied topological cyclic homology from a couple of viewpoints. I addressed the natural question of what the $\mathbf{A}^1$-localization of TC is, i.e., “homotopy TC”. Unlike Weibel’s homotopy K-theory, which is still a very interesting invariant of schemes and agrees with K-theory on smooth schemes, I was able to prove that this invariant is always zero in characteristic p and purely rational integrally [Elm21]. One can interpret my vanishing result as saying that $\mathbf{A}^1$-homotopy theory is extremely incompatible with trace methods.

Another viewpoint is that trace methods should be used to study not just nilpotent extensions of rings but also of categories. With Sosnilo, I was able to formulate and prove a generalization of the theorem of Dundas-Goodwillie-McCarthy [Goo86, Dun97, McC97, DGM13] relating the K-theory of a nilpotent extension to its TC to the setting of so-called **weighted categories** of Bondarko [Bon10]. In [ES20] we used the theory of weights to reduce the problem to nilpotent extensions additive ∞-categories (which we also defined); our results apply to nilpotent extension of algebraic stacks as well as to the functor from Voevodsky’s category of motives to classical Chow motives.

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