GAGA TYPE CONJECTURE FOR THE BRAUER GROUP VIA DERIVED GEOMETRY

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In Brauer III, Grothendieck considered the problem of comparing the cohomological Brauer group $\mathrm{B}r(\mathrm{X}) = \mathrm{H}^2_{et}(\mathrm{X}, \mathrm{G}_m)$ of a scheme X, proper and flat over a henselian DVR R, and the inverse limit of the Brauer groups $\varprojlim_n \mathrm{B}r(\mathrm{X}_n)$, where $\mathrm{X}_n = \mathrm{X} \otimes_{\mathrm{R}} \mathrm{R}/m^n$. He proved that the canonical map $\mathrm{B}r(\mathrm{X}) \to \varprojlim_n \mathrm{B}r(\mathrm{X}_n)$ is injective under a number of restrictions, and left as an open problem the question on whether the formal injectivity holds in a fairly general setting. Thanks to the machinery of derived algebraic geometry and the results of Toën on derived Azumaya algebras and derived Morita theory, we are able to rephrase Grothendieck?s question in terms of a formal GAGA-type problem for smooth and proper categories, enriched over the ∞-category $\mathrm{QC}oh(\mathrm{X})$ of quasi-coherent O_{X}-modules. In this framework we can show that Grothendieck?s injectivity conjecture always holds for a proper derived scheme $\mathrm{X} \to \mathrm{S}$ where S is the spectrum of any complete Noetherian local ring, if we are willing to replace the inverse limit $\varprojlim_n \mathrm{B}r(\mathrm{X}_n)$ with the Brauer group $\mathrm{B}r(\mathfrak{X})$ of the formal scheme $\mathfrak{X}$ given by the colimit of the thickenings X_n. The obstruction involving the inverse system $\mathrm{P}ic(\mathrm{X}_n)$ considered by Grothendieck appears naturally in the Milnor sequence for a certain tower of spaces. This is a joint work in progress with Mauro Porta (IRMA, Strasbourg).