COUNTS OF RATIONAL CURVES ON DEL PEZZO SURFACES ENRICHED IN BILINEAR FORMS

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Del Pezzo surfaces here are smooth projective surfaces with ample anticanonical bundle, including $\mathbb{P}^2, \mathbb{P}^1 x \mathbb{P}^1$, and cubic surfaces. By imposing the condition that a rational curve of fixed degree passes through an appropriate number of points, the number of such curves is finite. Over the complex numbers, these counts are independent of the generic choice of points. This invariance of number fails over the reals, but there is a beautiful method of Welschinger to correct this. It is a feature of $\mathbb{A}^1$-homotopy theory that analogous real and complex results can indicate the presence of a common generalization, valid over a general field. For $\mathbb{A}^1$-connected Del Pezzo surfaces under appropriate hypotheses, we give counts of rational curves valued in the group completion GW(k) of symmetric, non-degenerate, bilinear forms over k, which are again independent of the generic choice of points. By replacing the positive integer count with such a bilinear form, one records information about the field of definition of the rational curve and the tangent directions at its nodes. We compute some low degree examples, including on the Del Pezzo surfaces listed above. This is joint work with Jesse Kass, Marc Levine, and Jake Solomon.